Spin-Torsion in Chaotic Inflation

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Abstract

The role of spin-torsion coupling to gravity is analyzed in the context of a model of chaotic inflation. The system of equations constructed from the Einstein-Cartan and inflaton field equations are studied and it is shown that spin-torsion interactions are effective only at the very first e-folds of inflation, becoming quickly negligible and, therefore, not affecting the standard inflationary scenario nor the density perturbations spectrum predictions. PACS number(s): 98.80 Cq

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Inflation, in its many different implementations, has become one of the most important cosmological paradigm today [for reviews, see for instance, [1]]. The underlying idea of inflation, of a period of accelerated expansion of the scale factor, when the energy density is dominated by a vacuum energy density, is able to provide in a simple way a solution to the cosmological horizon and flatness problems and at the same time provides a model for density perturbations in the early Universe. Earlier studies by Gasperini [2] have shown that inflation could be driven by a spin density dominated epoch in the early Universe, even in the absence of vacuum dominant contributions to the energy density, showing that a spin-torsion interaction acts like a source of repulsive gravity. This then poses us with the question whether primordial spin-torsion interactions are able to support inflation in standard inflaton driven inflationary scenarios, by, e.g., easing the conditions for slow-roll of the inflaton field. Previous works on spin/torsion effects in inflation that we are aware of [3] have not detailed or elucidated the real role of spin-torsion in an inflationary epoch. Torsion makes an important role in very different physical models [4, 5]. In particular, torsion is natural to many models of higher dimensional theory, as in Einstein-Kalb-Ramond models and string theory [6] and in gauge theories of the Poincarè group [7]. Therefore, it is natural to expect that torsion may be particularly important in pre-inflationary models, where quantum gravity effects may be introduced, from the geometrical aspects of the space-time, by a torsion interaction term. This may be the case in chaotic inflationary scenarios, where the inflaton initial conditions are taken around the Planck era and, then, quantum gravity effects may become important to determine the initial conditions prior to inflation. Based on the above motivations, in this letter, by considering the spin-spin interactions of matter as described by the Einstein-Cartan theory (see, *e.g.*, Ref. [7]), we study the role of spin-torsion in the simplest model of chaotic inflation, which is that of an inflaton with a quadratic potential. We do not expect that more general models of chaotic inflation will lead to results much different to this simple model, when regarding the effects of spin-torsion, which is introduced through a generalization of the gravity equations. In the Einstein-Cartan theory, the gravity equations are modified such that the Friedman equation (we are assuming a spatially flat Friedman-Robertson-Walker metric) and the acceleration equations read [2], respectively,

$$H^2 = \frac{8\pi G}{3}(\rho_\phi + \rho_{\rm s}) \tag{1}$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_\phi + 3p_\phi - 8\pi G\rho_{\rm s}) \,, \tag{2}$$

where $H = \dot{a}/a$ is the Hubble parameter and $G = 1/M_{\rm pl}^2$, with $M_{\rm pl}$ the Planck mass. In the above equations we have also defined $\rho_{\rm s}$, as $\rho_{\rm s} = \langle S_{\mu\nu\alpha}S^{\mu\nu\alpha}\rangle/2$, the average of the square of the spin density tensor $S_{\mu\nu\alpha}$. The spins are taken as randomly oriented (from not polarized spinning matter fields) [2], so the average value of S is zero. The torsion tensor $Q_{\mu\nu}^{\ \ \alpha}$ is related with $S_{\mu\nu}^{\ \ \alpha}$ by the standard expression [7]

$$Q_{\mu\nu}^{\ \ \alpha} = 8\pi G \left(S_{\mu\nu}^{\ \ \alpha} + \frac{1}{2}\delta_\mu^\alpha S_{\nu\beta}^{\ \ \beta} - \frac{1}{2}\delta_\nu^\alpha S_{\mu\beta}^{\ \ \beta} \right) . \tag{3}$$

In Eqs. (1) and (2), ρ_ϕ and p_ϕ are the energy density and pressure for the inflaton field ϕ, respectively, given by the usual relations: $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$, $p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$. As the spin-torsion does not couple to the inflaton field, we have the evolution equation for ϕ,

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 , \tag{4}$$

where in the above equations overdot represent derivative with respect to time and $V' = \frac{dV}{d\phi}$ is the field derivative of the inflaton potential. Using the simplest inflaton potential for chaotic inflation, $V(\phi) = m^2\phi^2/2$, and from Eqs. (1), (2) and (4), we determine the system of equations satisfying the scale factor a, ϕ and $\rho_{\rm s}$,

$$\begin{aligned} \ddot{a} &= -\frac{8\pi G}{3} a \left(\dot{\phi}^2 - \frac{m^2}{2}\phi^2 - 4\pi G \rho_{\rm s} \right) \\ \ddot{\phi} &= -3\frac{\dot{a}}{a}\dot{\phi} - m^2\phi \\ \dot{\rho}_{\rm s} &= -6\frac{\dot{a}}{a}\rho_{\rm s} . \end{aligned} \tag{5}$$

From the equation for the acceleration in the set of equations in (5), we can immediately see that a spin-torsion density works in favor of inflation, when the slow-roll conditions for the inflaton fields applies. However, under these circumstances, of a regime of inflation, the Universe quickly enters in a de Sitter phase, with $H \sim$ const. and, therefore, from the last of the equations in (5), $\rho_{\rm s}$ satisfies

$$\rho_{\rm s} \sim e^{-6Ht} \quad \text{(in de Sitter)} \tag{6}$$

decreasing with the sixth power of the inverse of the scale factor during the de Sitter phase and then the spin density quickly vanishes as soon the Universe enters in an inflationary phase. Note that $\rho_{\rm s}$ decreases much faster than the inflaton density (and even faster than radiation energy densities), which goes with the third power of the inverse of the scale factor during the de Sitter phase. Therefore, we do not find a spin-torsion dominated inflation over the inflaton in chaotic inflation models in general, with spin-torsion interactions very fast becoming subdominat right after the first e-folds of inflation. This is consistent with earlier findings from Kao in [3], which working in the context of torsion in the ten-dimensional Kalb-Ramond theory, concludes that the torsion field vanishes at the end of the inflationary era. For the same reasons above, we do not expect any contribution of spin-torsion interactions to density perturbations in chaotic inflationary scenarios, once the spin-torsion density is depleted well before quantum fluctuations first cross the horizon. We can ask what happens when the initial value for ϕ is much smaller than the usual value needed in the model above in the absence of spin-torsion effects, $\phi_i \sim 3.4 M_{\rm pl}$, as required for sufficiently inflation (~ 70 e-folds of inflation) in the model. It seems that, from the first of the equations in (5), we could arrange a spin-torsion dominated epoch over the inflaton field, for a sufficiently large initial value for $\rho_{\rm s}$. However, this seems not to be the case, since, from Eq. (1), it imposes a limit on the initial value for $\rho_{\rm s}$, as $2\pi G \rho_{{\rm s},i}$ cannot be larger than $\rho_{\phi,i}$. Also, very specific models as the one discussed by Gasperini in [2], shows that a spin-torsion dominated inflation, with the physical requirements of large enough e-foldings of inflation, can only be attained if we require a extreme fine-tuning for the spin density prescription used in there.

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